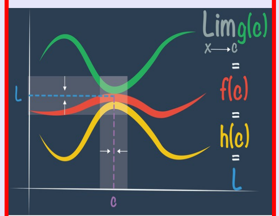
Calculus I

Lecture 8



Feb 19-8:47 AM

$f'(x)$ evaluated at $x = a$ is $f'(a)$

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

for $f(x) = x^3$

$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{x^3 - a^3}{x - a}$$

$$= \lim_{x \to a} \frac{\cancel{(x-a)}(x^2 + xa + a^2)}{\cancel{x-a}}$$

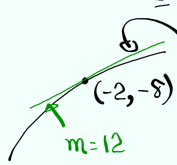
$$= \lim_{x \to a} (x^2 + xa + a^2) = a^2 + a \cdot a + a^2$$

$$= a^2 + a^2 + a^2 = \boxed{3a^2}$$

If $a = -2$

$f(-2) = (-2)^3 = -8$

$f'(-2) = 3(-2)^2 = 12$



$y - (-8) = 12(x - (-2))$

$y + 8 = 12(x + 2)$

$\boxed{y = 12x + 16}$

Jun 30-8:02 AM

For $f(x)=\sqrt{x}$, find $f'(4)$.

$$f'(4)=\lim_{x\to 4}\frac{f(x)-f(4)}{x-4}=\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$$

$f(4)=\sqrt{4}=2$

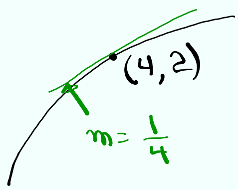
$(A-B)(A+B)=A^2-B^2$

$$=\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}$$

$$=\lim_{x\to 4}\frac{(\sqrt{x})^2-(2)^2}{(x-4)(\sqrt{x}+2)}$$

$$=\lim_{x\to 4}\frac{\cancel{x-4}^{\,1}}{\cancel{(x-4)}(\sqrt{x}+2)}$$

$$=\lim_{x\to 4}\frac{1}{\sqrt{x}+2}=\frac{1}{\sqrt{4}+2}=\boxed{\frac{1}{4}}$$



$y-2=\frac{1}{4}(x-4)$

$\boxed{y=\frac{1}{4}x+1}$

Jun 30-8:09 AM

Given $y=f(x)$

First derivative y' or $f'(x)$

y-Prime f-prime of x

New notation

$f'(x)=\frac{d}{dx}[f(x)]$, $y'=\frac{dy}{dx}$

Prove $\frac{d}{dx}[x^4]=\boxed{4x^3}$ ← $f'(x)$, y'

$f(x)=x^4$ $y=x^4$

$$\frac{d}{dx}[x^4]=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$ where $f(x)=x^4$

$$=\lim_{h\to 0}\frac{(x+h)^4-x^4}{h}$$

Bionomial expansion

$$=\lim_{h\to 0}\frac{\cancel{x^4}+4x^3h+6x^2h^2+4xh^3+h^4\cancel{-x^4}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(4x^3+6x^2h+4xh^2+h^3)}{\cancel{h}}$$

$$=\lim_{h\to 0}(4x^3+\cancel{6x^2h}^{\to 0}+\cancel{4xh^2}^{\to 0}+\cancel{h^3}^{\to 0})=\boxed{4x^3}$$

Jun 30-8:15 AM

Prove $\frac{d}{dx}[x^n] = n x^{n-1}$ $\qquad \frac{d}{dx}[x^4] = 4x^{4-1} = 4x^3$

$$\frac{d}{dx}[\overset{f(x)}{x^n}] = \lim_{h \to 0} \frac{\boxed{(x+h)^n}^{f(x+h)} - \boxed{x^n}^{f(x)}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^n} + n x^{n-1} h + \frac{n(n-1)}{2} x^{n-2} h^2 + \cdots + h^n - \cancel{x^n}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left(n x^{n-1} + \frac{n(n-1)}{2} x^{n-2} h + \cdots + h^{n-1}\right)}{\cancel{h}}$$

$$= \lim_{h \to 0} \left[n x^{n-1} + \frac{n(n-1)}{2} x^{n-2} h + \cdots + h^{n-1} \right]$$

$\to 0 \quad \to 0 \quad \to 0 \quad \to 0$

$$= \boxed{n x^{n-1}}$$

$f(x) = x^5$

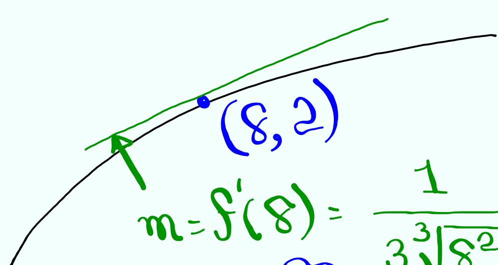
$f'(x) = 5x^{5-1} = \boxed{5x^4}$

$f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -1x^{-1-1} = -1x^{-2} = \boxed{\frac{-1}{x^2}}$

Jun 30-8:25 AM

Find eqn of the Tan. line to the graph of $f(x) = \sqrt[3]{x}$ at $x = 8$. $\qquad f(8) = \sqrt[3]{8} = 2$



$y - 2 = \frac{1}{12}(x - 8)$ $\qquad y = \frac{1}{12}x - \frac{2}{3} + 2$

$y = \frac{1}{12}x - \frac{8}{12} + 2$ $\qquad \boxed{y = \frac{1}{12}x + \frac{4}{3}}$

$m = f'(8) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{3\sqrt[3]{64}} = \frac{1}{3 \cdot 4} = \frac{1}{12}$

$f(x) = \sqrt[3]{x} = x^{1/3}$

$f'(x) = \frac{1}{3} x^{\frac{1}{3} - 1} = \frac{1}{3} x^{-2/3} = \frac{1}{3x^{2/3}} = \frac{1}{3\sqrt[3]{x^2}}$

Jun 30-8:34 AM

Prove $\frac{d}{dx}\left[\sin x\right] = \cos x$

$f(x) = \sin x$ $\quad$ $f'(x) = \cos x$

$y = \sin x$ $\quad$ $y' = \cos x$

$\sin(A+B) = \sin A\cos B + \cos A \sin B$

$$\frac{d}{dx}\left[\sin x\right] = \lim_{h\to 0} \frac{\overset{f(x+h)}{\boxed{\sin(x+h)}} - \overset{f(x)}{\boxed{\sin x}}}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cosh + \cos x \sinh - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\boxed{\sin x \cosh - \sin x} + \cos x \sinh}{h}$$

$$= \lim_{h\to 0} \frac{\sin x(\cosh - 1) + \cos x \sinh}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sin x(\cosh - 1)}{h} + \frac{\cos x \sinh}{h}\right]$$

$$= \lim_{h\to 0} \frac{\sin x(\cosh - 1)}{h} + \lim_{h\to 0}\frac{\cos x \sinh}{h}$$

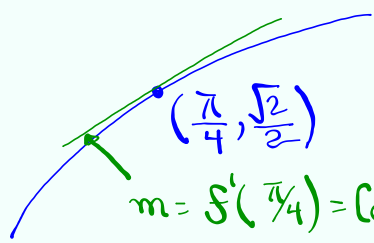
$$= \sin x \cdot \underbrace{\lim_{h\to 0} \frac{\cosh - 1}{h}}_{\to 0} + \cos x \cdot \underbrace{\lim_{h\to 0}\frac{\sinh}{h}}_{\to 1}$$

$$= \cancel{\sin x \cdot 0}^{\,0} + \cos x \cdot 1 = \boxed{\cos x}$$

Jun 30-8:43 AM

find eqn of the tan. line to the graph of $f(x) = \sin x$ at $x = \frac{\pi}{4}$.

$f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$



$m = f'(\pi/4) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$f(x) = \sin x$

$f'(x) = \cos x$

$y - y_1 = m(x - x_1)$

$y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$$\boxed{y = \frac{\sqrt{2}}{2}x - \frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2}}$$

Jun 30-8:54 AM

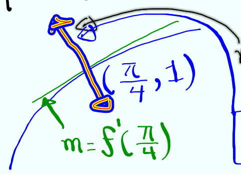
$\frac{d}{dx}[\sin x] = \cos x$ $\qquad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[\cos x] = -\sin x$ $\qquad$ $\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\tan x] = \sec^2 x$ $\qquad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$

Find the slope of the normal line to the graph $f(x) = \tan x$ at $x = \frac{\pi}{4}$.

$\tan\frac{\pi}{4} = 1$



$m = \dfrac{-1}{f'(\frac{\pi}{4})}$

$\boxed{m = \frac{-1}{2}}$ slope of the normal line at $x = \pi/4$.

$f(x) = \tan x$

$f'(x) = \sec^2 x$

$f'(\frac{\pi}{4}) = \sec^2\frac{\pi}{4} = \left(\frac{1}{\cos\pi/4}\right)^2 = \left(\frac{1}{\frac{\sqrt{2}}{2}}\right)^2 = \left(\frac{2}{\sqrt{2}}\right)^2 = \frac{4}{2} = \boxed{2}$

Jun 30-8:59 AM